分析与偏微分方程青年学者研讨会

为了交流国内分析与偏微分方程领域的最新研究成果，增强彼此之间的了解与合作，同时为国内分析与偏微分方程领域的青年学者提供一个学术交流研讨的平台，复旦大学、天津大学以及北京应用物理与计算数学研究所将于 2023 年 6 月 22 日至 26 日在复旦大学举办“分析与偏微分方程青年学者研讨会”，其中 6 月 22 日（周四）注册报到，6 月 26 日（周一）离会。本会议得到国家重点研发计划青年科学家项目（2021YFA1002500）和国家自然科学基金的资助。

组织者：陈　曦　　复旦大学
贺丹青　　复旦大学
李康伟　　天津大学
郑继强　　北京应用物理与计算数学研究所

其他注意事项

1. 进校：进校前请按下图流程**提前登记**（会议地点在**江湾校区**，登记时也可以一并选上**邯郸校区**），方便及时进校

2. WIFI：eduroam，用户名：apde2023@guest，密码：Apde2023
3. 交通：
   A. 本次会议的住宿地点（也是晚餐、晚宴地点）为**宝隆宾馆**（上海市虹口区逸仙路180 号），部分老师和同学的住宿在**宝丰联**（距离宝隆步行 1 分钟）。上述酒店离上海站较近。乘坐地铁 3 号线在大柏树或江湾镇下车，然后步行约 1 公里。
   B. 本次会议报告地点在复旦大学江湾校区**上海数学中心谷超豪报告厅**，地址为：上海市杨浦区淞沪路 2005 号
   C. 会议通勤车辆上午发车时间为上午 **8:20（宝隆宾馆），下午 17：50（数学中心）**. 请大家提前规划好时间，以免错过。

日程表

	6 月 22 日	6 月 23 日	6 月 24 日	6 月 25 日	6 月 26 日
09：00-09：50		开幕式	高传伟（郑继强）	张　城（贺丹青）	
09：50-10：30		茶歇	茶歇	茶歇	
10：30-11：20		曹　军（陈　曦）	耿　俊（李康伟）	刘博辰（贺丹青）	
11：20-12：10		陈　鹏（陈　曦）	郭少明（李康伟）	张瑞祥（贺丹青）	
12：10		午餐	午餐	午餐	
14：00-14：50	注册	陈祥宏（郑继强）	李文娟（李康伟）		离会
14：50-15：30		茶歇	茶歇		
15：30-16：20		程　星（郑继强）	汝少雷（李康伟）	自由讨论	
16：20-17：10		杨建伟（郑继强）	席亚昆（贺丹青）		
17：10-17：40		李雪梅（陈　曦）	龚禹霖（陈　曦）		
18：30		晚餐	晚宴		

报告信息

	6 月 23 日	主持人
09：00-09：50	开幕式	
09：50-10：30	茶歇	
10：30-11：20	曹　军（浙江工业大学） Heat kernel and wavelets on metric measure space	陈　曦
11：20-12：10	陈　鹏（中山大学） The Garnett--Jones Theorem on BMO spaces associated with operators and applications	陈　曦
12：10	午餐	
14：00-14：50	陈祥宏（中山大学） Spacetime estimates for the semiperiodic Schrödinger equation	郑继强
14：50-15：30	茶歇	
15：30-16：20	程　星（河海大学） Global solution of the Euler-Poisson equation	郑继强
16：20-17：10	杨建伟（北京理工大学） Strichartz estimates for wave equations with a potential in non-trapping exterior domains	郑继强
17：10-17：40	李雪梅（北京师范大学） Dynamics of the focusing, energy-critical Hartree equation with radial data	陈　曦
18：30	晚餐	

	6 月 24 日	主持人
09：00-09：50	高传伟（首都师范大学） Some problems in harmonic analysis of curved settings	郑继强
09：50-10：30	茶歇	
10：30-11：20	耿 俊（兰州大学） Hardy spaces and the Neumann problem in L^p for elliptic equation with periodic high-contrast coefficients in Lipschitz Domains	李康伟
11：20-12：10	郭少明（威斯康星大学麦迪逊分校） 平面上的极大 Radon 变换	李康伟
12：10	午餐	
14：00-14：50	李文娟（西北工业大学） On L^{p}-improving bounds for maximal operators associated with curves of finite type in the plane	李康伟
14：50-15：30	茶歇	
15：30-16：20	汝少雷（浙江师范大学） Lp′-Lp estimates for several classes of dispersive semi-group	李康伟
16：20-17：10	席亚昆（浙江大学） Can you hear your location on a manifold?	贺丹青
17：10-17：40	龚禹霖（清华大学） Spectral Distribution of Twisted Laplacian on Typical Hyperbolic Surfaces of High Genus	陈 曦
18：30	晚宴	

	6 月 25 日	主持人
09：00-09：50	张　城（清华大学） Sharp Lp estimates and size of nodal sets of generalized Steklov eigenfunctions	贺丹青
09：50-10：30	茶歇	
10：30-11：20	刘博辰（南方科技大学） Existence of Fourier series on Euclidean subsets	贺丹青
11：20-12：10	张瑞祥（加州大学伯克利分校） A new conjecture to unify Fourier restriction and Bochner-Riesz	贺丹青
12：10	午餐	
14：00-14：50	自由讨论	
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18：30		

曹军

题目：Heat kernel and wavelets on metric measure space

摘要：The heat kernel is a universal tool that plays a dominant role in mathematics and physics due to its simple and powerful properties. The construction of heat kernel is the starting point for conducting a detailed analysis of a general metric measure space. In this talk, we provide a new analytic mthod to construct heat kernel via wavelts on such setting. The obtained heat kernel enjoys many good properties including the stochastic completeness property, the two-sided stable-like estimate and the Hölder continuity estimate. This is a joint work with Profs. A. Grigor'yan and L. Liu.

陈鹏

题目：The Garnett--Jones Theorem on BMO spaces associated with operators and applications

摘要：Let X be a metric space with doubling measure, and L be a nonnegative self-adjoint operator on $L^2(X)$ whose heat kernel satisfies the Gaussian upper bound.
In this talk, we give a construction that for every $ f$ in the $ {\rm BMO}_L(X)$ space associated with the operator L, we have comparable upper and lower bounds for the distance $$ {\rm dist} (f, L^{\infty}):= \inf_{g\in L^{\infty}} \|f -g\|_{{\rm BMO}_L(X)}$$ by the constant in the John and Nirenberg inequality for the space ${\rm BMO}_L(X)$ space: $$ \sup_B { \mu\{ x\in B: |f(x)-e^{-{r_B^2}L}f(x)|>\lambda\} \over \mu(B)} \leq e^{-\lambda/\varepsilon}\ \ \ \ {\rm for\ large\ } \lambda,$$ which extends the theorem of Garnett and John \cite{GJ1} for the classical BMO space of John and Nirenberg. We also give a construction that a compact supported $\BMO_L(X)$ function can be decomposed as the summation of a L^∞-function and the integral of the heat kernel with respect to a finite Carleson measure. The key new technique is a geometric construction involving the semigroup e^{-tL}. We also resort to several fundamental tools including the stopping time argument and the random dyadic lattice.

陈祥宏

题目: Spacetime estimates for the semiperiodic Schrödinger equation

摘要: We will first review some spacetime estimates for the Schrödinger equation, such as Strichartz estimates and local-smoothing type estimates, for both nonperiodic and periodic solutions in R^n. Then we will discuss similarities and differences that appear in the case of semiperiodic solutions (i.e., solutions periodic in part of the space variables). This is joint work with Zihua Guo (Monash), Minxing Shen (SYSU) and Lixin Yan (SYSU).

程星

题目：Global solution of the Euler-Poisson equation

摘要：We consider the compressible Euler-Poisson equations for polytropes. Firstly, we develop two variational problem for γ = 4/3 and γ ∈(6/5, 4/3) respectively. The first variational problem for γ = 4/3 is related to the best constant of a Hardy-Littlewood type inequality. The best constant obtained is sharp and it yields a threshold of the mass to the gaseous star. For γ ∈(6/5,4/3), we construct a type of constraint variational problem attained by the Lane-Emden function. Then, we show that the spherically symmetric finite energy weak solution globally exists if the mass is less than the Chandrasekhar limit mass for γ = 4/3 or the initial data belongs to an invariant set constructed by the variational argument for γ ∈(6/5,4/3). Furthermore, we conditionally obtain that the support of the gaseous star expands as time tends to infinity with a virial argument.

高传伟

题目：Some problems in harmonic analysis of curved settings

摘要：Restriction theory and its related problems,such as the Bochner-Riesz means, Kakeya, and local smoothing estimate are extensively studied. These studies also promote other problems in PDEs and geometric theory, etc. In this talk, we introduce the curved variants of the above problems, for example, the oscillatory integral operator (curved restriction) and the curved Kakeya. In the curved setting,some new phenomena will happen, in particular the Kakeya compression, which says the curved Kakeya set may be concentrated near a submanifold. We will show how that influences the result in the curved settings.

耿俊

题目：Hardy spaces and the Neumann problem in L^p for elliptic equation with periodic high-contrast coefficients in Lipschitz Domains

摘要：We establish the optimal L^p estimates for the Neumann and regularity problems in the homogenization of elliptic equations with rapidly oscillating and high-contrast coefficients in

perforated Lipschitz domains in $\mathbb{R}^d$ for $$1<p<2+\alpha.$$ All the ranges of $p's$ are sharp.

龚禹霖

题目: Spectral Distribution of Twisted Laplacian on Typical Hyperbolic Surfaces of High Genus

摘要: We investigate the spectral distribution of the twisted Laplacian associated with uniform square-integrable bounded harmonic 1-form on a typical hyperbolic surface of high genus. First, we estimate the spectral distribution by the supremum norm of the corresponding harmonic form. Subsequently, we show that the square-integrable bounded harmonic form exhibits a small supremum norm for a typical hyperbolic surface of high genus. Based on these findings, we prove a uniform Weyl law for the distribution of real parts of the spectrum on a typical hyperbolic surface.

郭少明

题目：平面上的极大 Radon 变换

摘要：报告的内容是关于平面上的某些极大 Radon 变换的有界性，例如 Bourgain 的圆极大算子，以及沿着椭圆的极大算子等等。证明有界性的主要工具是傅里叶 decoupling 不等式。

李文娟

题目：On L^{p}-improving bounds for maximal operators associated with curves of finite type in the plane

摘要： In this paper, we study the L^{p}-improving property for the maximal operators along a large class of curves of finite type in the plane with dilation set $E \subset [1,2]$. The L^{p}-improving region depends on the order of finite type and the fractal dimension of E. In particular, various impacts of non-isotropic dilations are also deeply considered. This is a jointed work with Dr. Huiju Wang.

李雪梅

题目: Dynamics of the focusing, energy-critical Hartree equation with radial data

摘要: We study the long time dynamics of the radial solutions for the focusing, energy-critical Hartree equation. The nondegeneracy of the positive bubble solutions plays a key role in the spectral analysis of the linearized operator, the construction of the special threshold solutions, and the classification of the threshold solutions. The main arguments are the spectral theory of the linearized operator, the modulational analysis and the concentration compactness rigidity argument pineered by T. Duyckaerts, F. Merle and C. Kenig to classify the threshold solutions for the energy critical NLS and NLW. This is joint work with C. Liu, X. Tang and G. Xu.

刘博辰

题目: Existence of Fourier series on Euclidean subsets

摘要: Fourier series is a very powerful tool in nature. In this talk we will introduce different types of Fourier basis, such as orthogonal basis, Riesz basis, frames, etc., and discuss about their existence on Euclidean subsets. We will also say something about its relation to tiling.

汝少雷

题目 ：Lp′-Lp estimates for several classes of dispersive semi-group

摘要 ：Strichartz estimates play an important role in the study of well-posedness of initial value problems, large-time behavior of solutions and regularity of solutions. The Lp′-Lp estimate of dispersive semi-groups is one of the important tools for studying Strichartz estimates. Van der Corput lemma is commonly used in the proof for these problems, and which was extended by the ”Stationary set” estimate theorem derived by Zhang et al. in 2021. In this paper, by using ”Stationary Set” estimation theorem, we will discuss the Lp′-Lp estimates for several classes of dispersive semi-group.

席亚昆

题目: Can you hear your location on a manifold?

摘要: We introduce a variation on Kac's question, "Can one hear the shape of a drum?" Instead of trying to identify a compact manifold and its metric via its Laplace--Beltrami spectrum, we ask if it is possible to uniquely identify a point x on the manifold, up to symmetry, from its pointwise Weyl counting function. This problem has a physical interpretation. You are placed at an arbitrary location in a familiar room with your eyes closed. Can you identify your location in the room by clapping your hands once and listening to the resulting echos and reverberations?

Our main result provides an affirmative answer to this question for a generic class of metrics.

杨建伟

题目：Strichartz estimates for wave equations with a potential in non-trapping exterior domains

摘要: Consider the linear wave equations with a potential outside a compact set with a non-trapping boundary condition. Assuming that the potential satisfies an appropriate decreasing condition and that zero is not an eigenvlue or a resonance, we prove that the solution of the wave equation projected to the subspace associated to the continuous spectrum of the Schrödinger operator satisfies global Strichartz estimates. Our method is based on the local Strichartz estimate in exterior domains without potential, a result going back to Smith-Sogge, and the local energy decay which is obtained from a uniform resolvent estimate. This work is joint with Thomas Duyckaerts.

张城

题目： Sharp Lp estimates and size of nodal sets of generalized Steklov eigenfunctions

摘要： We investigate a generalized Steklov problem with a non-smooth potential on the boundary. We establish sharp L^p estimates for the Steklov eigenfunctions on compact manifolds with boundary, using their L^2 norms on the boundary. To do this, we show L^p bounds for the harmonic extension operators and the spectral projection operators on the boundary by establishing new heat kernel bounds and resolvent estimates for the Dirichlet-to-Neumann operator with a non-smooth potential. These global L^p estimates are new even in the absence of potential, and related to the pointwise estimates by Polterovich-Sher-Toth and Galkowski-Toth. As an application, we obtain a lower bound on the size of the boundary nodal sets, which extends the results of Wang-Zhu to generalized Steklov eigenfunctions.

张瑞祥

题目： A new conjecture to unify Fourier restriction and Bochner-Riesz

摘要：The Fourier restriction conjecture and the Bochner-Riesz conjecture ask for Lebesgue space mapping properties of certain oscillatory integral operators. They both are central in harmonic analysis, are open in dimensions ≥ 3, and notably have the same conjectured exponents. In the 1970s, H\"{o}rmander asked if a more general class of operators (known as H\"{o}rmander type operators) all satisfy the same L^p-boundedness as in the above two conjectures. A positive answer to H\"{o}rmander's question would resolve the above two conjectures and have more applications such as in the manifold setting. Unfortunately H\"{o}rmander's question is known to fail in all dimensions ≥ 3 by the work of Bourgain and many others. It continues to fail in all dimensions ≥ 3 even if one adds a ``positive

curvature'' assumption which one does have in restriction and Bochner-Riesz settings. Bourgain showed that in dimension 3 one always has the failure unless a derivative condition is satisfied everywhere. Joint with Shaoming Guo and Hong Wang, we generalize this condition to arbitrary dimension and call it ``Bourgain's condition''. We unify Fourier restriction and Bochner-Riesz by conjecturing that any H\"{o}rmander type operator satisfying Bourgain's condition should have the same L^p-boundedness as in those two conjectures. As evidences, we prove that the failure of Bourgain's condition immediately implies the failure of such an L^p-boundedness in every dimension. We also prove that current techniques on the two conjectures apply equally well in our conjecture and make some progress on our conjecture that consequently improves the two conjectures in higher dimensions. I will talk about some history and some interesting components in our proof.